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A SYSTEMATIC APPROACH TO THE BOW PROBLEM.(U)

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Nabil Daoud

This research was carried out under the
Naval Sea Systems Command
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20. Abstract (continued)

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ABSTRACT

The wave making effectiveness of a ship is an important feature that must be fully recognized in developing a uniformly valid slender-ship theory for the wave resistance problem. The usual results of ordinary slender-ship theory are invalid near the bow of a slender ship because the theory fails to take into account the dominant effects of the divergent wave system in this region. A new approach based on the matched asymptotic expansions technique is used to solve the bow problem. The general solution of the far-field problem includes the well-known wave-free potentials plus the potential due to a line distribution of wave sources that extends outside the limits of the ship to guarantee the proper description of the far-field flow. The bow-near-field problem and the general form of its solution are derived. In order to match the two solutions in their overlap domain near the bow, the far-field source function is decomposed into two parts that include a "slowly" varying source function and a "rapidly" varying source function. The slowly varying sources can be determined from the solution of the bow-near-field problem. However, the rapidly varying sources, which are associated with the disturbances ahead of the bow and consequently needed to obtain the solution of the bow-near-field problem, cannot be determined unless we match the bow flow to the middle-body flow. The next step in the analysis will concern this problem.

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1. INTRODUCTION

The present investigation concerns the steady forward motion of a ship on the free surface of an inviscid fluid. The exact formulation of the problem is well-known (see, for example, Wehausen 1973) but generally intractable, because of the nonlinearity of the free surface conditions. No mathematical progress is possible unless the problem can be linearized, for which purpose additional simplifying assumptions must be made.

Typical ship hulls are long and slender, which is a logical feature to exploit in the theoretical investigation of such problems. The first clear attempt towards a slender ship theory for the case of steady forward motion was made by Cummins (1956). Unfortunately, he did not obtain any firm results that could be confirmed experimentally, and his work was forgotten for many years. Subsequent investigations of the problem were carried out independently by Vossers (1962), Maruo (1962), and Tuck (1963).

Vossers reduced the linearized boundary-value problem to an integral equation for the potential on the ship using Green's theorem. Then he evaluated the asymptotic expansions of the integrals in a region near to the slender-ship and obtained a first approximation for the potential in this region. Vossers approach requires difficult approximations of multiple integrals that discouraged further developments.

Tuck used the powerful technique of matched asymptotic expansions in his investigations. He matched the inner and outer expansions in an "intermediate" limiting region and obtained a first approximation for the potential in this region. His results are essentially similar to Vosser's approximation. However, Tuck has made rigorous estimates of the error involved in his approximations and established the required conditions for the validity of the results. In particular, he found that the sectional area curve must have an absolutely integrable first derivative, a continuous second derivative and a piecewise continuous third derivative.

It is rather obvious that ship hull forms do not satisfy these stringent requirements, and Tuck postulated that the conditions can be relaxed if the results are interpreted as approximations rather than limiting values. In particular, he assumed that discontinuities in the second derivatives of the sectional area curve could be tolerated. This assumption was taken for granted in subsequent applications of the theory, even though Tuck noted that

"the complexity of the argument needed even to prove sufficiency (of the conditions) suggests that relaxation of the above conditions is unlikely." In the next section, we will show that the full force of Tuck's conditions are needed in order to justify the validty of his results.

Maruo investigated the wave resistance of a slender ship by evaluating the asymptotic expansion of Michell's integral when the draft becomes small and of the same order as the beam. He obtained the same resistance formula as Vossers and Tuck found, but with some additional end-effect terms.

Although Vossers and Maruo did not state any explicit conditions for the validity of their procedures, the similarity between their results and Tuck's results suggest the possibility that their analyses are subject to the same conditions obtained by Tuck. In fact, the results of these different theories become identical if the above stated conditions are satisfied.

Unfortunately, Maruo's work has caused a major set back in the development of the theory, because it led to the belief that slender-ship theory results can be considered as a special case of thin-ship theory. This conclusion is based mainly on the assumption that Maruo's analysis is valid for all ship forms. There is no doubt that Maruo's results are acceptable if the above stated restrictions on the hull form are fulfilled. But for general hull forms, we may use simple arguments to show that Maruo's analysis becomes questionable altogether. From purely mathematical considerations, the asymptotic approximation of an integral when one of its parameters becomes small should differ from the exact integral by an error term that becomes small when this parameter is small. However, it turned out that predictions of the wave resistance from Michell's integral differ appreciably from Maruo's asymptotic formula for ordinary hull forms with small drafts (see, for example, Kotik and Thomsen 1963). In fact, the asymptotic formula predicts negative wave resistance at the low and high speed limits! This simply means that Maruo's results cannot possibly represent the asymptotic behavior of Michell integral for ordinary hull forms. Actually, the shortcomings of Maruo's analysis can be attributed to the singular limits of the treated integrals as was shown by Ursell (1960). We may conclude now that the above statement concerning the relation between slender-ship theory and thin-ship theory is basically unfounded for ordinary hull forms.

Apparently, existing slender-ship theory for the wave resistance does not correspond to the physical problem of interest. Because actual ship hull forms are not "smooth" enough to fulfill the stringent requirements of the theory, especially, near to the ends. For this reason, investigations of the end-effects rendered necessary in order to overcome the shortcomings of the theory. In this respect, Maruo's end effect terms are considered incorrect here, because we believe that his analysis is wrong if the ends are not smooth in the first place.

The first successful attempt towards a study of the end-effects was made by Ogilvie (1972). He assumed that, near the bow, gradients in the longitudinal direction are still small compared to transverse derivatives, but not to the same extent as slender-ship theory assumes. He considered the particular case in which the resulting boundary-value problem for the flow in the near field satisfies the two dimensional Laplace equation in the transverse planes, together with the "full" linearized free surface condition that is familiar from classical thin-ship theory. In order to obtain a unique solution for the problem without additional information from the far-field, Ogilvie assumed furthermore that all disturbances vanish ahead of the bow and at large side-way distances from the body. He solved the problem for the special case of a fine wedgelike bow by satisfying the body boundary condition on its centerplane. Daoud (1975) extended the solution to the case when the body boundary condition is satisfied on the exact body surface.

Perhaps the most encouraging results concerning slender-ship theory are those obtained by Ogilvie (1972). He compared predictions from his theory with measured wave heights along the surface of a wedgelike bow and obtained satisfactory agreements. However, the theory is valid only in a small region near the bow and it ignores completely the interaction effects between the flow in this region and everywhere else.

Thus, it is necessary to incorporate Ogilvie's bow-flow problem, which is basically a near-field problem, into the framework of a "uniform" slender-ship theory for the following two reasons. First, the theory will enable us to determine the correct far-field source distribution in the bow region so that we may obtain more realistic predictions of the wave resistance. The second reason concerns the solution of the bow-near-field problem, where the theory can provide an estimate on the initial disturbance at the bow and the appropriate radiation condition in the transverse planes.

The present investigation concerns the development of a "uniform" slender-ship theory for the case of steady forward motion. The term "uniform" implies here that the theory takes into account the influence of the "end effects", especially at the ship's bow. However, it is important to realize that the end effects associated with slender-ship theory are conceptually different from those arising from slender-bodies in unbounded fluids. Generally, the end effects in unbounded fluids are due mainly to violation of the slenderness assumption, as in the case of a blunt nose or a stagnation point flow (see Van Dyke, 1964). But in slender-ship theory, those effects are more closely related to the nature of the wave system produced by the ship on the free surface. Details of this behavior will be discussed in the next section.

Apparently, the wave making effectiveness of a slender ship is a distinguishing feature that must be fully recognized in developing the theory. For this reason, some of the established techniques of slender-bodies in unbounded fluids could lead to the wrong conclusions if applied to ship problems. In particular, representing the far-field flow of a ship by a line of appropriate singularities along its axis does not guarantee a true description of the wave system generated by the actual ship. The general form of the far-field solution is discussed in Section 3.

2. FAILURE OF ORDINARY SLENDER-SHIP THEORY

It has long been known that experimental confirmation of slender-ship theory results for the wave resistance are even less encouraging than for Michell's integral. This has led to the belief that, even for ships of small draft-length ratio, the vertical distribution of sources represented by Michell's integral is important. The failure of the theory has therefore been attributed to the fact that the far-field sources are located right at the undisturbed free surface.

The wave making effectiveness of a line distribution of sources placed at the free surface was studied by Ogilvie (1977). He considered the special case of a line distribution that gives the same wave resistance as Michell's integral. When the analysis was applied to the case of a strutlike body, the resulting source function seems to smooth out the discontinuous behavior predicted by ordinary slender-ship theory at the corners of the body. The smoothing effect takes place over a relatively short region and the resulting source function extends outside the limits of the body. Nevertheless, the wave resistance which can be computed from the source distribution depends critically on that small amount of rounding off predicted by the analysis.

Now one might argue that the shortcomings of slender-ship theory cannot be blamed on placing the far-field sources at the free surface, but rather on the wrong prediction of their distribution. However, this is an over-simplification of the problem, since the difficulty is more closely related to the peculiarities of the flow caused by a line distribution which is insufficiently smooth. We note that this problem is not pronounced as much in thin-ship theory, because the sources are distributed over the centerplane of the ship and integration in the vertical direction alleviates the effects of any discontinuities.

In order to understand the nature of the difficulties associated with slender-ship theory, we considered the flow caused by a line of sources placed on the free surface in Appendix I. The investigation of the Appendix concerns mainly the asymptotic behavior of the flow near to the singular line. A brief discussion on the results of the Appendix is presented here.

The velocity potential for this problem was written in a form which separates the local disturbance type of behavior from the wave effects. The

asymptotic approximation of the local disturbance terms is independent on the smoothness of the source distribution, but requires the source function to be continuous. On the other hand, the behavior of the wave term near to the singular line is more complicated and highly dependent on the smoothness of the source function. This term was treated in such a way that enabled us to distinguish between the transverse and divergent wave systems. It was found that if the source function has a continuous first derivative the contribution of the divergent wave system becomes negligible near to the singular line, and the usual results of ordinary slender-ship theory are valid, as expected in this case. However, if the first derivative of the source function is discontinuous at some point x_0, the contribution of the divergent wave system dominates the flow field in a small region where $(x-x_0)=O(|y|^{1/2})$, but it becomes negligible again when $(x-x_0)=O(1)$.

It is now obvious that the full force of Tuck's conditions as discussed in the introduction are needed in order to ensure the validity of his results. But it is unlikely that actual ship hull forms can satisfy the stringent requirements of the theory, especially at the ship bow. Therefore, the failure of the theory can be attributed simply to the fact that it does not correspond to the physical problem of interest. In particular, the theory fails because it does not take into account the dominant effects of the divergent-wave system in the bow region. For this reason, we expect the theory to give unreasonable predictions of the divergent wave system, since it ignores their effects in the first place.

These conclusions may be confirmed by investigating the results reproduced in Figure 1 from Tsutsumi's paper (1978). The Figure shows a comparison between the measured wave spectrum of the model with predictions from Michell theory and slender-ship theory. The sectional area curve of the model has a continuous first derivative everywhere that tends to zero at the ends, but it can be shown that the second derivative of the curve has a square root singularity at both ends. Thus the requirements of ordinary slender-ship theory are not fulfilled, although its predictions are bounded because the source distribution is continuous. According to our previous discussion, we expect the theory to give wrong predictions of the divergent wave system, and that is exactly what the Figure shows.

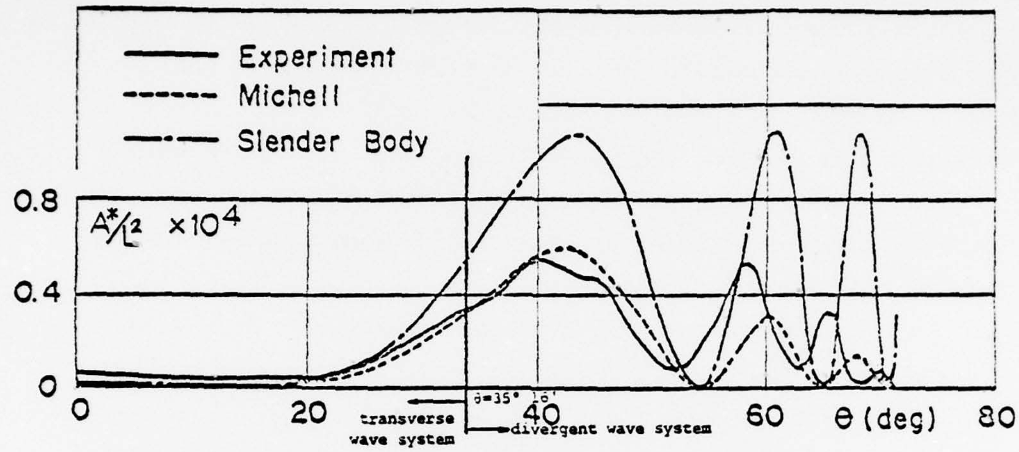

Figure 1. Wave spectra of M.No. 331 at Fn = 0.319
(from Tsutsumi 1978)

Moreover, the Figure indicates that predictions of the divergent-wave spectrum from Michell theory and slender-ship theory have similar behaviors (their extremes occur at the same points) which deviate appreciably from the behavior of the experimental results at large values of θ. This suggests the possibility that Michell's theory suffers the same ills of slender-ship theory, but to a lesser degree because of the vertical distribution of sources. However, generalization of this conclusion is beyond the scope of the present work.

According to the results of Appendix I, the effects of the divergent-wave system in the near-field are confined to a small region of $O(\varepsilon^{1/2})$ along the ship, where ε is the slenderness parameter. In principle, this region is centered at points along the hull where the conditions of ordinary slender-ship theory are not fulfilled. However, these conditions are generally violated at the bow of actual ships and the term "bow flow" will be used here to designate the nature of the flow in such regions. The subsequent investigation deals mainly with the solution of the problem at the ship's

bow, but the analysis can also be applied to other points along the hull after some modifications.

In the bow region, the asymptotic behavior of the far-field solution is characterized by the parameter $\kappa x^2/|y| = O(1)$, where x represents distances measured along the ship from the bow and y represents transverse distances. The appropriate inner expansion may be obtained in this case by approaching the body along these characteristic lines (see Cole 1968). Thus in the bow-near-field problem, where $y = O(\varepsilon)$, we must require that $\sqrt{\kappa}x=O(\varepsilon^{1/2})$ in order to maintain the same character of the flow in this region. Ogilvie (1972) used physical reasoning to obtain these requirements in his study of the bow flow problem.

However, as we proceed along the body and x becomes $O(1)$, the divergent waves will move off to the far-field and the transverse waves dominate the flow field near the body. The term "middle-body flow" will be used to distinguish this type of behavior in which the usual assumptions of ordinary slender-ship theory are valid.

It is now obvious that a uniform slender-ship theory must include the above mentioned two near-field problems. Each problem should be matched separately to the appropriate asymptotic approximation of the far-field. However, to complete the solution of the problem we must match the bow flow to the middle body flow.

3. THE FAR-FIELD PROBLEM AND ITS SOLUTION

We shall use a coordinate system, Oxyz, with its origin fixed to the bow of the ship. The plane Oxy coincides with the mean free surface, and the z-axis is vertically upward. The ship is travelling with a constant speed U in the negative x-direction.

As usual in slender-body theory, we infer the existence of a small parameter ε, which is a measure of the "slenderness" of the ship. This parameter may be simply the ratio of the maximum transverse dimension of the ship to its length, or it may be the maximum slope with respect to the longitudinal axis of planes tangent to the ship's surface. It is assumed here that the characteristic length of the transverse waves produced by the ship is comparable to the ship's length L, i.e., the parameter $U^2/g = O(1)$ (we also consider that $L = O(1)$).

The complete outer expansion of the exact problem is given by Tuck (1963). Accordingly the first order term in the outer expansion of the velocity potential satisfies Laplace's equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z}\right) \Phi(x,y,z) = 0 \quad , \tag{3.1}$$

in the fluid domain. The boundary condition on the mean free surface $z=0$ is

$$\Phi_{xx} + \kappa\Phi_z = 0 \quad , \tag{3.2}$$

where $\kappa = g/U^2$. The potential Φ must also satisfy the appropriate radiation condition, which is essentially a requirement that the waves lie behind the disturbance.

In the far-field, the ship shrinks down to a line and the hull boundary condition disappears from the problem. Since there are no other disturbing agents in the fluid, the potential must be analytic everywhere in the lower half-plane except for possible singularities on this line. For this reason, it is usually assumed that the far-field solution can be obtained by a distribution of appropriate singularities along the axis of the body.

This procedure can easily be justified for slender bodies in unbounded fluids. The general form of the far-field solution in this case can be expressed in terms of distributions of sources and multipoles along the x-axis (Ogilvie 1977, p. 100). These distributions are singular along the x-axis

and therefore cannot be permitted outside the limits of the body.

However, it is not evident that a distribution of wave singularities along the axis of the ship could produce the actual wave system in the far field of a slender ship. In fact, Ogilvie (1977) has shown that the line of sources which can generate the far field flow of a strutlike body and can produce the same wave system as predicted by thin-ship theory must extend along the entire x-axis. Thus in order to guarantee that the far-field solution will present accurately the wave-making effectiveness of the actual ship, it is essential to distribute the wave singularities along the entire x-axis. But this requirement contradicts our previous assertion that the potential must be analytic everywhere in the fluid domain except on the axis of the ship. Therefore, the far-field solution is determined according to the following rule:

> "we shall require the solution of the far-field problem to include the most general form of solutions to the boundary-value problem *provided* that the asymptotic expansion of these solutions near to the x-axis is non-singular outside the axis of the ship."

Accordingly, we shall construct the velocity potential in the far field by the superposition of potential functions each satisfying indentically all the boundary conditions appropriate to the far-field problem. Thus we can write

$$\Phi(x,y,z) = \phi(x,y,z) + \sum_{n=0}^{\infty} \Phi_n(x,y,z) \quad , \tag{3.3}$$

where ϕ represents the potential due to a line distribution of Kelvin sources along the x-axis, and the terms Φ_n represent wave-free potentials. The form of the potentials Φ_n can easily be obtained from the work of Ursell (1968), thus

$$\Phi_n = \cos(n\theta)\int_{-\infty}^{\infty} dk\, e^{-ikx} \quad a_n^*(k)\; K_n(|k|r) \quad , \tag{3.4}$$

where r,θ are the polar coordinates in cross-planes perpendicular to the x-axis, such that, $y = r \sin\theta$ and $z = -r\cos\theta$. Here, K_n is the modified Bessel function of the second kind as defined by Abramowitz and Stegun (1964). The functions $a_n^*(k)$ represent the Fourier transforms of

the unknown coefficients $a_n(x)$ in the expansion. These functions must satisfy the relation

$$a^*_{2n+1} - 2\frac{|k|}{\kappa} a^*_{2n} + a^*_{2n-1} = 0 \quad , \tag{3.5}$$

with $a^*_{-1}=0$. It can easily be shown that the wave-free potentials satisfy Laplace's equation (3.1) and the free surface condition (3.2), while the radiation condition is satisfied trivially.

It is convenient to use a Fourier transform representation of the potential for the line of Kelvin sources. One such form is given by Tuck (1963) as

$$\phi(x,y,z) = -2\kappa \int_{-\infty}^{\infty} dk\, e^{-ikx}\, m^*(k) \cdot$$

$$\lim_{\mu\to 0} \int_{-\infty}^{\infty} d\lambda \frac{\exp[-i\lambda y + z\sqrt{\lambda^2+k^2}]}{\kappa\sqrt{\lambda^2+k^2} - (k + i\mu/2)^2} \quad , \tag{3.6}$$

where μ is the so-called dissipation factor of Rayleigh. Here, $m^*(k)$ represents the Fourier transform of the source function $m(x)$,

$$m^*(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dx\, e^{ikx}\, m(x) \quad .$$

It will be assumed that $m(x)$ is continuous with piecewise continuous first derivative and absolutely integrable. This assumption is needed for the analysis of the next section.

It is preferable at this point to let $\mu\to 0$ in (3.6) and deform the path of integration in the cut λ-plane into the proper indented contour $C(k)$ as shown in Figures 2a, 2b, and 2c according to the value of k. The poles of the integrand are now at the points $\lambda=\pm\lambda_0$, where

$$\lambda_0 = \frac{|k|}{\kappa}\sqrt{k^2-\kappa^2} \quad , \quad \text{for } |k| > \kappa \quad ;$$

$$\lambda_0 = i\frac{|k|}{\kappa}\sqrt{\kappa^2-k^2} \quad , \quad \text{for } |k| < \kappa \quad .$$

The expression for ϕ will now be put in a more useful form. For $y>0$, the contour $C(k)$ may be completed as indicated in Figure 2. Since

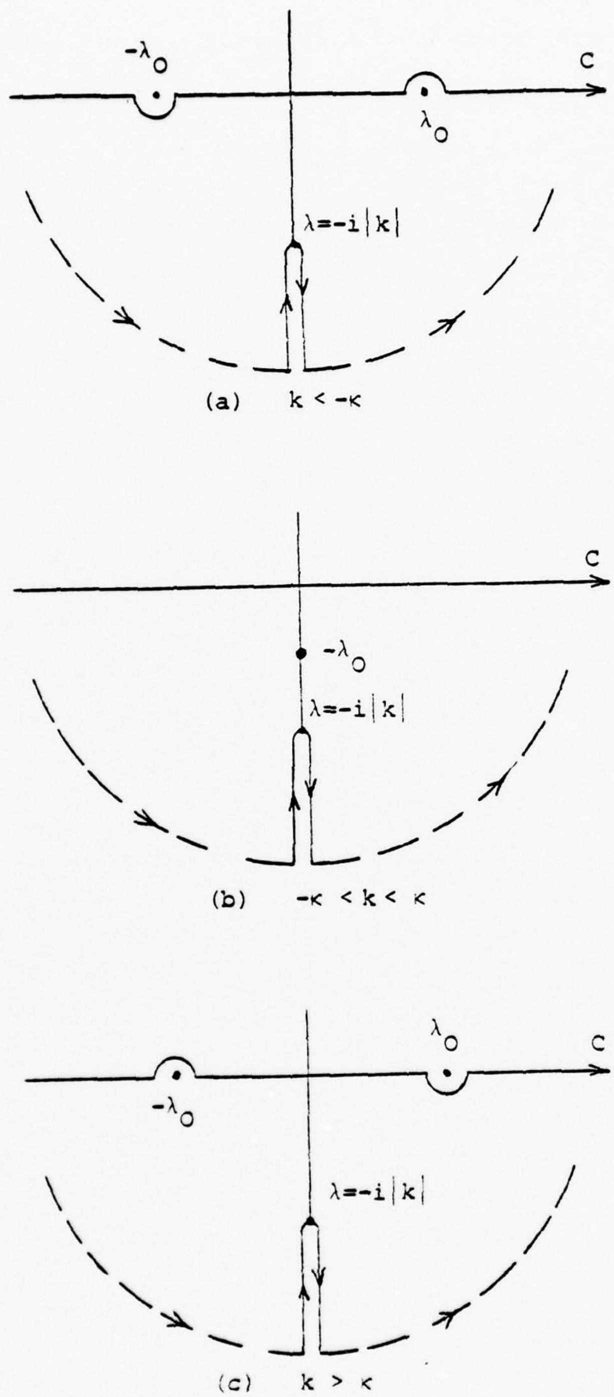

Figure 2. Contours in the cut λ-plane

the physical problem is symmetric with respect to y, we need to consider only this case ($y>0$) and set $y=|y|$ in the final results so that they will be valid for all y. The integral along the dotted semi-circle can be made as small as desired by taking the radius R sufficiently large. Therefore its contribution vanishes as $R\to\infty$.

The potential ϕ can now be expressed as the sum of two parts, namely

$$\phi = \phi_1 + \phi_2 ,$$

where ϕ_1 represents the outcome of the integration along the two sides of the branch cut shown in Figure 2, and ϕ_2 is the contribution from the residues. Specifically, we get

$$\phi_1 = -4\kappa \int_{-\infty}^{\infty} dk\, e^{-ikx}\, m^*(k) \cdot$$

$$\mathrm{Re} \int_{|k|}^{\infty} du\, \frac{\exp[-r(u \sin|\theta| + i\sqrt{u^2-k^2} \cos\theta)]}{\kappa\sqrt{u^2-k^2} + i\, k^2} \; ; \tag{3.7}$$

and

$$\phi_2 = -4\pi i \int_{-\infty}^{\infty} \frac{k\, dk}{\sqrt{k^2-\kappa^2}}\, e^{-ikx}\, m^*(k) \cdot$$

$$\exp\left[-\frac{r}{\kappa} (k^2 \cos\theta - ik\sqrt{k^2-\kappa^2} \sin|\theta|)\right] , \tag{3.8}$$

where the above integral should be taken along the upper-side of the branch cut between $k=\pm\kappa$.

4. ASYMPTOTIC EXPANSION OF THE FAR-FIELD POTENTIAL NEAR THE BOW

In this section we shall investigate the behavior of the potential Φ for points close to the bow. As was explained in section 2, the approximation that is of interest here is one in which $r = O(\varepsilon)$ and $|x| = O(\varepsilon^{1/2})$.

Let us consider first the asymptotic behavior of the potential ϕ_1 as given by (3.7). Substitute $u = |k|\sqrt{\lambda^2+r}/\sqrt{r}$ in the integral with respect to u in (3.7), then change k to $\ell/\sqrt{r}$ afterwards, hence

$$\phi_1 = -\frac{4\kappa}{\sqrt{r}} \int_{-\infty}^{\infty} d\ell\, e^{-i\ell x/\sqrt{r}}\, m^*(\ell/\sqrt{r})\, \mathrm{Re} \int_0^{\infty} \frac{\lambda d\lambda}{\sqrt{\lambda^2+r}\,(\kappa\lambda+i|\ell|)}$$

$$\exp\left[-|\ell|(\sqrt{\lambda^2+r}\,\sin|\theta| + i\,\lambda\cos\theta)\right] . \tag{4.1}$$

This expression is now in a suitable form for the intended approximation in which the parameter $x/\sqrt{r}$ can be kept fixed, i.e., $x/\sqrt{r} = O(1)$. At present, the function $m^*(\ell/\sqrt{r})$ is assumed fixed. Therefore, we can set $r=0$ in the integral over λ and obtain the first approximation for ϕ_1 in the form

$$\phi_1 = -\frac{4\kappa}{\sqrt{r}} \int_{-\infty}^{\infty} d\ell\, e^{-i\ell x/\sqrt{r}}\, m^*(\ell/\sqrt{r}) \cdot$$

$$\mathrm{Re} \int_0^{\infty} d\lambda\, \frac{\exp(-i|\ell|\lambda e^{-i|\theta|})}{\kappa\lambda + i|\ell|} + E_1 \ , \tag{4.2}$$

where E_1 denotes the error involved in this approximation. The bounds of this error term are estimated in Appendix II, part A, and it shows that $E_1 = O(\varepsilon^{1/2})$.

In the expression (4.2), we complete the contour of integration with respect to λ as shown in Figure 3 and use the residue theorem to evaluate the integral, thus

$$\phi_1 \sim -\frac{4\kappa}{\sqrt{r}} \int_{-\infty}^{\infty} d\ell\, e^{-i\ell x/\sqrt{r}}\, m^*(\ell/\sqrt{r}) \cdot \int_0^{\infty} \frac{d\lambda}{\kappa\lambda-|\ell|}\, e^{-|\ell|\lambda\cos\theta} \cos(|\ell|\lambda\sin|\theta|)$$

$$-\frac{4\pi\kappa}{\sqrt{r}} \int_{-\infty}^{\infty} d\ell\, e^{-i\ell\kappa x/\sqrt{r}}\, m^*(\kappa\ell/\sqrt{r})\, e^{-\kappa\ell^2\cos\theta} \sin(\kappa\ell^2\sin|\theta|) \ , \tag{4.3}$$

where the inner integral of the first term is interpreted as a Cauchy principal value integral.

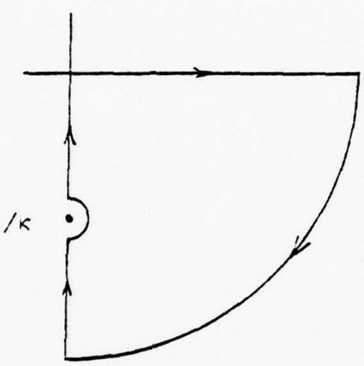

Figure 3. Contour of integration in the λ-plane

Since $m(x)$ is assumed continuous with piecewise continuous first derivative, we can use the convolution theorem to evaluate the first term in the above expression (Bochner 1959, p. 59). After the supstitution $\lambda = u^2/|\ell|$ in the integral with respect to λ we evaluate the ℓ-integration first, getting

$$\phi_1 \sim -\frac{4\kappa}{\sqrt{r}} \int_{-\infty}^{\infty} d\xi\, m(\xi) \int_0^{\infty} du\, e^{-\kappa u^2 \cos\theta} \cos(\kappa u^2 \sin|\theta|) \sin(\kappa u|x-\xi|/\sqrt{r}) -$$

$$-\frac{4\pi\kappa}{\sqrt{r}} \int_{-\infty}^{\infty} d\ell\, e^{-i\ell\kappa x/\sqrt{r}}\, m^*(\kappa\ell/\sqrt{r})\, e^{-\kappa\ell^2\cos\theta} \sin(\kappa\ell^2 \sin|\theta|) \,. \tag{4.4}$$

Let us consider next the asymptotic approximation of the potential ϕ_2 which is given by equation (3.8). After the substitution $\ell = \sqrt{r}\, k/\kappa$, the expression for ϕ_2 becomes

$$\phi_2 = -\frac{4\pi\kappa i}{\sqrt{r}} \int_{-\infty}^{\infty} \frac{\ell d\ell}{\sqrt{\ell^2-r}} e^{-i\ell\kappa x/\sqrt{r}} \quad m^*(\kappa\ell/\sqrt{r})$$

$$\exp\left[-\kappa(\ell^2\cos\theta - i\ \ell\sqrt{\ell^2-r}\ \sin|\theta|\right] \quad , \tag{4.5}$$

where the integral must be taken along the upper side of the branch cut between $\ell=\pm\sqrt{r}$.

Again the above expression can be approximated simply by setting $r=0$ while keeping $x/\sqrt{r}$ finite. Thus we get

$$\phi_2 = -\frac{4\pi\kappa i}{\sqrt{r}} \int_{-\infty}^{\infty} d\ell\ e^{-i\ell\kappa x/\sqrt{r}} \quad m^*(\kappa\ell/\sqrt{r})\ \mathrm{sgn}(\ell)$$

$$\exp\left[-\kappa(\ell^2\cos\theta - i\ \ell|\ell|\sin|\theta|\right] + E_2 \quad , \tag{4.6}$$

where E_2 is the error involved in this approximation. We have also shown in Appendix II, Part B, that $E_2 = 0(\varepsilon^{1/2})$.

The approximation of the wave potential ϕ in the bow region can be obtained by adding (4.4) and (4.6), therefore

$$\phi \sim -\frac{4\kappa}{\sqrt{r}} \int_{-\infty}^{\infty} d\xi\ m(\xi) \int_0^{\infty} du\ e^{-\kappa u^2\cos\theta} \cos(\kappa u^2\sin\theta)\ \sin(\kappa u|x-\xi|/\sqrt{r}) -$$

$$-\frac{4\pi\kappa i}{\sqrt{r}} \int_{-\infty}^{\infty} d\ell\ e^{-i\ell\kappa x/\sqrt{r}} \quad m^*(\kappa\ell/\sqrt{r})\ \mathrm{sgn}(\ell)\ e^{-\kappa\ell^2\cos\theta}\cos(\kappa\ell^2\mathrm{sgn}\theta) \ . \tag{4.7}$$

By using the convolution theorem on the second term in (4.7), we get

$$\phi \sim -\frac{4\kappa}{\sqrt{r}} \int_{-\infty}^{\infty} d\xi\ m(\xi) \int_0^{\infty} du\ e^{-\kappa u^2\cos\theta} \cos(\kappa u^2\sin\theta)\ \sin(\kappa u|x-\xi|/\sqrt{r}) -$$

$$-\frac{4\kappa}{\sqrt{r}} \int_{-\infty}^{\infty} d\xi\ m(\xi) \int_0^{\infty} du\ e^{-\kappa u^2\cos\theta} \cos(\kappa u^2\sin\theta)\ \sin(\kappa u(x-\xi)/\sqrt{r}) \ . \tag{4.8}$$

Although the two terms in the above expression can easily be combined into a single term with the integral over ξ taken between the limits $\xi=-\infty$ and $\xi=x$, it is more convenient to treat them separately at this stage.

In fact, the first term in (4.8) behaves like $(\log r)$ as $r \to 0$ and therefore it is singular along the entire x-axis (this behavior can be obtained by using the Fourier transform representation of this term as given by (4.3) and expressing the principal value integral in terms of the exponential integral E_i as defined by Abramowitz and Stegun (1965)), while the second term is well behaved along the x-axis, as can be shown from (4.7). Therefore equation (4.8) cannot be used in its present form to approximate the potential ahead of the bow unless we can remove the singular behavior of its first term.

We shall suppose now that the source function $m(x)$ is composed of two parts, namely

$$m(x) = m_S(x) + m_R(x) \quad , \tag{4.9}$$

where $m_S(x)$ is a "slowly varying" function in the sense that differentiation keeps orders of magnitude unchanged; while $m_R(x)$ is a "rapidly varying" non oscillating function with the understanding that differentiation increases orders of magnitude by $\varepsilon^{-1/2}$, i.e., $m_R'(x) = O(m_R \varepsilon^{-1/2})$. The physical interpretation of this decomposition can be explained as follows: The function $m_S(x)$ represents sources that are associated with longitudinal variation of the ship form and therefore its derivative is proportional to the rate of change of the hull form in the x-direction which is O(1). It is also expected that m_S will vanish outside the limits of the ship. On the other hand, $m_R(x)$ is associated with disturbances caused in the bow region and hence it will be confined to a region of $O(\varepsilon^{1/2})$.

The potential ϕ can be written now as the sum of two parts, namely

$$\phi = \phi_S + \phi_R \quad ,$$

where ϕ_S and ϕ_R represent the contributions of the sources m_S and m_R respectively to the right hand side of (4.8). Thus

$$\phi_S = -\frac{8\kappa}{\sqrt{r}} \int_0^x d\xi \; m_S(\xi) \int_0^\infty du \; e^{-\kappa u^2 \cos\theta} \cos(\kappa u^2 \sin\theta) \, \sin(\kappa u(x-\xi)/\sqrt{r}) \; . \tag{4.10}$$

In the above expression, we have assumed that $m_S(x)$ is identically zero when $x<0$ in order to remove the singular behavior of (4.8) ahead of the

bow. Consequently, the functions m_S and m_R may be discontinuous at $x=0$, even though their sum is continuous everywhere.

Let us consider now the potential ϕ_R, which may be written as

$$\phi_R = 2\sum_{n=0}^{\infty} \frac{(-)^{n+1}}{(2n+1)!}\left(\frac{\kappa^2}{r}\right)^{n+1} \int_{-\infty}^{\infty} d\xi\ |x-\xi|^{2n+1}\ m_R(\xi)$$

$$\int_0^{\infty} du\ u^n\ e^{-\kappa u\ \cos\theta}\ \cos(\kappa u\ \sin\theta)\ -$$

$$-\ \frac{4\kappa}{\sqrt{r}} \int_{-\infty}^{\infty} d\xi\ m_R(\xi) \int_0^{\infty} du\ e^{-\kappa u^2\cos\theta}\ \cos(\kappa u^2\sin\theta)\ \sin(\kappa u(x-\xi)/\sqrt{r})\ , \tag{4.11}$$

where the series was obtained by expanding the sine function in the first term of (4.8). Consider the integral with respect to ξ in the first term of the above expression; if we write

$$M_n(x) = \int_{-\infty}^{\infty} d\xi\ |x-\xi|^{2n+1}\ m_R(\xi)\quad ,$$

then it can be easily shown that

$$\frac{d^{2n+2}}{dx^{2n+2}} M_n = 2\ (2n+1)!\ \ m_R(x)\quad . \tag{4.12}$$

We can use (4.12) to write $M_n(x)$ in the form of a Fourier transform representation. Hence

$$M_n(x) = 2(-)^{n+1}\ (2n+1)! \int_{-\infty}^{\infty} dk\ e^{-ikx}\ \frac{m_R^*(k)}{k^{2n+2}}\quad . \tag{4.13}$$

After the substitution $k=u/\sqrt{r}$ in (4.13), we can use it to express (4.11) in the form

$$\phi_R = \frac{4}{\sqrt{r}} \sum_{n=1}^{\infty} (n-1)!\ \cos(n\theta) \int_{-\infty}^{\infty} du\ e^{-iux/\sqrt{r}}\ m_R^*(u/\sqrt{r})\ \left(\frac{\kappa}{u^2}\right)^n\ -$$

$$- \frac{4\pi\kappa i}{\sqrt{r}} \int_{-\infty}^{\infty} du \; e^{-i\kappa u x/\sqrt{r}} \; m_R^*(\kappa u/\sqrt{r}) \; \mathrm{sgn}(u) \; e^{-\kappa u^2 \cos\theta} \; \cos(\kappa u^2 \sin\theta) \; . \tag{4.14}$$

The last term is here written in the same form as the second term in (4.7), which is equivalent to the last term of (4.11). Although the first term (the sum) of the above expression is still singular ahead of the bow, we will show that this behavior can be cancelled by the contribution of the wave-free potentials.

Let us consider the asymptotic expansion of the wave-free potentials in the bow region. From (3.3) and (3.4), we can write these potentials in the form

$$\phi_0 \equiv \sum_{n=0}^{\infty} \cos(n\theta) \int_{-\infty}^{\infty} dk \; e^{-ikx} \; b_n^*(k) \; (\kappa/2|k|)^n \; K_n(|k|r) \quad , \tag{4.15}$$

in which we replaced the unknown functions a_n^* by $(\kappa/2|k|)^n \; b_n^*$. Now the relation (3.5) becomes

$$\left(\frac{\kappa}{2|k|}\right)^2 b_{2n+1}^* - b_{2n}^* + b_{2n-1}^* = 0 \quad . \tag{4.16}$$

In the expression (4.15), substitute $\ell=\sqrt{r}\,k$ and then make use of (4.16) and the well-known approximations of the Bessel K-functions for small arguments. We obtain

$$\phi_0 \sim -\sqrt{r} \int_{-\infty}^{\infty} d\ell \; e^{-i\ell x/\sqrt{r}} \; b_1^*(\ell/\sqrt{r}) \; (\kappa/2|\ell|)^2 \; \log(|\ell|\sqrt{r})$$

$$+ \frac{1}{2\sqrt{r}} \sum_{n=1}^{\infty} (2n-2)! \; \cos(2n-1)\theta \int_{-\infty}^{\infty} d\ell \; e^{-i\ell x/\sqrt{r}} \; b_{2n-1}^*(\ell/\sqrt{r}) \cdot \left(\frac{\kappa}{\ell^2}\right)^{2n-1}$$

$$+ \frac{1}{2\sqrt{r}} \sum_{n=1}^{\infty} (2n)! \; \cos 2n\theta \int_{-\infty}^{\infty} d\ell \; e^{-i\ell x/\sqrt{r}} \; \left(\frac{\kappa}{\ell^2}\right)^{2n} \cdot$$

$$\left[b_{2n-1}^*(\ell/\sqrt{r}) + r(\kappa/2|\ell|)^2 b_{2n+1}^*(\ell/\sqrt{r}) \right] \quad .$$

After neglecting the terms of $O(r^{1/2})$ in the above expression it becomes

$$\phi_0 \sim \frac{1}{2\sqrt{r}} \sum_{n=1}^{\infty} (n-1)! \cos n\theta \int_{-\infty}^{\infty} d\ell \; e^{-i\ell x/\sqrt{r}} \; b_n^*(\ell/\sqrt{r}) \left(\frac{\kappa}{\ell^2}\right)^n , \tag{4.17}$$

where the functions b_n^* satisfy now the relation

$$b_{2n}^* = b_{2n-1}^* \; . \tag{4.18}$$

We note that (4.18) represents now the approximation of (4.16) in the bow region.

It is evident at once from (4.14) and (4.17) that if we choose the unknown functions b_n^* such that

$$b_n^* = -8 \; m_R^* \; , \tag{4.19}$$

which satisfy (4.18) trivially, then the singular behavior of (4.14) will vanish.

Finally, we obtain the following expansion for the total potential Φ in the bow region

$$\Phi \sim -\frac{8\kappa}{\sqrt{r}} \int_0^x d\xi \; m_s(\xi) \int_0^{\infty} du \; e^{-\kappa u^2 \cos\theta} \cos(\kappa u^2 \sin\theta) \; \sin[\kappa u(x-\xi)/\sqrt{r}]$$

$$- \frac{4\pi\kappa i}{\sqrt{r}} \int_{-\infty}^{\infty} du \; e^{-i\kappa u x/\sqrt{r}} \; m_R^*(\kappa u/\sqrt{r}) \; \mathrm{sgn}(u) \; e^{-\kappa u^2 \cos\theta} \cos(\kappa u^2 \sin\theta) \; . \tag{4.20}$$

5. THE BOW NEAR-FIELD PROBLEM

As was explained in Section 2, the asymptotic approximation of the far-field flow near the bow is basically characterized by the parameter $\kappa x^2/|y|$, which is $O(1)$. This suggests that in the bow inner region, where $r=O(\varepsilon)$ we must require that $x = O(\varepsilon^{1/2})$, in order to maintain the same character of the flow.

Thus, the asymptotic expansion of the exact boundary-value problem in an inner region close to the bow can be obtained formally by assuming that

$$\frac{\partial f}{\partial y}\,,\ \frac{\partial f}{\partial z} = O(f\varepsilon^{-1})\ ,\ \frac{\partial f}{\partial x} = O(f\varepsilon^{-1/2}) \quad \text{as} \quad \varepsilon \to 0 \quad , \tag{5.1}$$

where f represents any flow variable in the near field. This statement means simply that derivatives in the longitudinal direction are still small compared to derivatives in the transverse direction but not to the same extent as ordinary slender-ship theory requires. The solution to be obtained will be valid in principle only in a region close to the bow, where $r = O(\varepsilon)$ and $x = O(\varepsilon^{1/2})$.

The first approximation for the potential ϕ (strictly, the first term in the asymptotic expansion) which is based on the above requirements was obtained by Ogilvie (1977). It satisfies Laplace's equation

$$\phi_{yy} + \phi_{zz} = 0 \quad , \tag{5.2}$$

in the fluid domain. The boundary condition on the undisturbed free surface $z=0$ is

$$\phi_{xx} + \kappa\phi_z = 0 \quad , \tag{5.3}$$

which is the same as the condition satisfied by the far-field problem. The body boundary condition becomes

$$\partial\phi/\partial N = -U\, n_1 \quad , \tag{5.4}$$

where $\underset{\sim}{n} = (n_1, n_2, n_3)$ is a unit vector normal to the hull surface and directed into the body; $\underset{\sim}{N}$ is a unit vector, lying in a crossplane, perpendicular to the contour of the body in that cross plane.

In order to solve the problem uniquely, Ogilvie assumed in addition that there is no disturbance present ahead of the bow and that $\phi = O(1/r)$ as

$r \to \infty$. These conditions were imposed so that the problem can be solved without the need of matching to a far-field solution. However, it is appropriate to assume here that any disturbance caused by the ship ahead of its bow will be confined to the bow inner region in which $|x| = O(\varepsilon^{1/2})$. Therefore, we will require the solution to satisfy the following conditions:

$$\lim_{x \to -\infty} \phi = 0 \quad ,$$

$$\lim_{x \to -\infty} \phi_x = 0 \quad . \tag{5.5}$$

The radiation condition which may be imposed on the solution as $r \to \infty$ can be determined from the far-field approximation of the previous section. However, we shall assume instead that $|\phi_y|$, $|\phi_x|$, and $|\phi_{xx}|$ are all bounded as $r \to \infty$, and seek a general solution for the problem.

6. SOLUTION OF THE BOW NEAR-FIELD PROBLEM AND ITS ASYMPTOTIC EXPANSION

The main purpose of this section is to obtain a solution for the boundary-value problem stated in the previous section. For general hull forms, the solution which satisfies the given boundary condition on the hull surface can only be determined by numerical methods. However, it is possible to represent the solution by a series expansion with unknown coefficients that can be determined later from the body boundary condition. The specific procedure for obtaining the coefficients of the series will not be discussed here, since we are interested at the present time in matching this solution to the far-field problem. The method which will be used to solve this problem is based largely on similar works by Ursell (1949, 1950, 1968).

It is convenient to introduce the complex Z-plane, which is defined as

$$Z = y + iz = - i\, r\, e^{-i\theta} \quad .$$

Consider the flow outside a circular cylinder, $|Z| = R$, that encloses the body and has its axis along the x-axis. Let

$$f(x;Z) = \phi(x;y,z) + i\, \psi(x;y,z)$$

be the complex velocity potential in this region (where ψ is the stream function conjugate to the velocity potential ϕ). The free surface condition (5.3) becomes

$$\mathrm{Re}\{f_{xx} + i\,\kappa\, f_Z\} = 0 \quad , \text{ on } \quad z = 0 \; . \tag{6.1}$$

The radiation conditions (5.5) are now in the form

$$\lim_{x\to-\infty} \mathrm{Re}\{f\} = 0 \quad ,$$
$$\lim_{x\to-\infty} \mathrm{Re}\{f_x\} = 0 \quad . \tag{6.2}$$

Also, from the assumed radiation conditions on ϕ as $r\to\infty$, we shall require that f be regular and that $|f_Z|$ and $|f_{xx}|$ be bounded outside $|Z| = R$.

Let us Fourier transform the complex potential f with respect to x, putting

$$f^*(\lambda;Z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x;Z)\, e^{j\lambda x}\, dx, \tag{6.3}$$

where we have used "j" as the imaginary unit to distinguish between the two complex planes used to solve the problem. When indicating "Re" and "Im" for the real and imaginary parts of a function, we will always suffix an i or j to the symbol to show whether we mean the real or imaginary part with respect to i or j [see Ogilvie and Tuck (1969) for more discussion]. The free surface condition (6.1) becomes

$$\mathrm{Im}_i\{f^*_Z + i\,\frac{\lambda^2}{\kappa}\,f^*\} = 0\ , \quad \text{on} \quad z = 0\ . \tag{6.4}$$

Let us now introduce the auxiliary function

$$G^*(\lambda;Z) = f^*_Z + i\,\frac{\lambda^2}{\kappa}\,f^*(\lambda;Z)\ , \tag{6.5}$$

which according to (6.4) satisfies the condition

$$\mathrm{Im}_i\ G^*(\lambda;y+i0) = 0\ . \tag{6.6}$$

By the Schwarz reflection principle, we extend G^* analytically into the upper half of the Z-plane

$$G^*(\lambda;y+iz) = \overline{G^*(\lambda;y-iz)}\ . \tag{6.7}$$

From the assumptions about f it follows that the extended function G^* is regular and bounded in the whole Z-plane outside $|Z| = R$. Thus G^* can be developed in a Laurent series (see Wehausen, 1974)

$$G^*(\lambda;Z) = C_0(\lambda) + \frac{C_1(\lambda)}{Z} + \frac{C_2(\lambda)}{Z^2} + \ldots\ , \quad |Z| > R\ . \tag{6.8}$$

The condition (6.6) implies that all the C_n must be real with respect to i.

We may now integrate the differential equation in f^* as given by (6.5) and obtain

$$f^*(\lambda;Z) = e^{-i\lambda^2 Z/\kappa} \int^{Z} d\xi\ \ G^*(\lambda;\xi)\ e^{i\lambda^2\xi/\kappa}\ .$$

The same value of f^* can be obtained by integration along two paths not enclosing the circle $|Z| = R$. However, integration along a closed path surrounding the circle $|Z| = R$ does not lead to a one-valued function due to the poles in (6.8) . Therefore, we introduce a cut in the complex Z-plane

extending from iR to $i\infty$ along the imaginary axis, across which analytic continuation is not permitted. Since $\phi^*=\mathrm{Re}_i\{f^*\}$ is an even function with respect to y it is continuous across the cut, while its conjugate ψ^* may be discontinuous. Denote by $\Delta\psi^*$ the discontinuity across the cut; since G^* is continuous across the cut, thus

$$\Delta\psi^*_z - \frac{\lambda^2}{\kappa}\Delta\psi^* = 0 \quad ,$$

and

$$\Delta\psi^* = B(\lambda)\ e^{\lambda^2 z/\kappa} \quad .$$

The objective now is to find a source function S which satisfies the boundary conditions of the problem and for which the Fourier transform S^* has the same discontinuity along the cut as f^*. Then the function $f^* - S^*$ will be continuous across the cut, and the cut may therefore be removed.

Consider the pseudo-impulsive source placed at the origin, as given by Wehausen and Laitone (1960, p 495)

$$S = -2\kappa \int_{-\infty}^{x} d\xi\ Q(\xi) \int_0^\infty \frac{dk}{\sqrt{\kappa k}}\ e^{-ikZ} \sin\sqrt{\kappa k}\ (x-\xi) \quad . \tag{6.9}$$

In which $Q(\xi)$ denotes the strength of the source. The Fourier transform of S may be written in the form

$$S^* = -Q^*(\lambda) \int_0^\infty \frac{e^{-ikZ}}{k-\lambda^2/\kappa}\ dk \quad , \tag{6.10}$$

where the integral is now taken along a path indented below the pole at $k=\lambda^2/\kappa$ [see Stoker (1956) p178 for detailed discussion]. After a somewhat lengthy transformation, it can be shown that S^* may be continued in the upper half of the Z-plane and that it has a discontinuity in its imaginary part with respect to i across the cut of magnitude

$$\mathrm{Im}_i\{\Delta S^*\} = 2\pi\ Q^*(\lambda)\ e^{\lambda^2 z/\kappa} \quad . \tag{6.11}$$

Consider the complex function

$$f^*(\lambda;Z) - S^*(\lambda;Z)$$

which is continuous across the cut if we set $Q^*(\lambda)=B(\lambda)/2\pi$ in (6.11); the cut therefore may be removed and the function has a Laurent expansion for $|Z|>R$. Thus

$$f^* - S^* = \sum_{n=0}^{\infty} q_n^*(\lambda)\ (iZ)^n + \sum_{n=1}^{\infty} q_{-n}^*(\lambda)\ (iZ)^{-n} = F_1^*(\lambda;Z) + F_2^*(\lambda;Z) \quad . \tag{6.12}$$

The coefficients q_n^* and q_{-n}^* are all real with respect to i, since ϕ^* is even.

It is possible to determine $F_1^*(\lambda;Z)$ explicitly. Using (6.10), we can write

$$S_Z^* + i\,\frac{\lambda^2}{\kappa}\,S^* = \frac{Q^*(\lambda)}{Z} \quad ,$$

from (6.12) and (6.8), we get

$$\frac{dF_1^*}{dZ} + i\,\frac{\lambda^2}{\kappa}\,F_1^* = C_0(\lambda) \quad .$$

Thus

$$F_1^*(\lambda;Z) = P^*(\lambda)\ e^{-i\lambda^2 Z/\kappa} - iC_0(\lambda)\kappa/\lambda^2 \quad , \tag{6.13}$$

where P^* is real with respect to i from the symmetry of ϕ^*.

Since both S^* and F_1^* satisfy the free surface condition separately, F_2^* must also satisfy (6.4) and it becomes

$$F_2^*(\lambda;Z) = \sum_{m=1}^{\infty} q_{2m}^*(\lambda)\{(iZ)^{-2m} + \frac{\lambda^2}{\kappa(2m-1)}\,(iZ)^{-2m+1}\} \quad . \tag{6.14}$$

Finally, we obtain the following expansion for f^* :

$$f^*(\lambda;Z) = S^*(\lambda;Z) + P^*(\lambda)\ e^{-i\lambda^2 Z/\kappa} + \sum_{m=1}^{\infty} q^*_{2m}(\lambda)\{(iZ)^{-2m} + \frac{\lambda^2}{\kappa(2m-1)}(iZ)^{-2m+1}\}. \tag{6.15}$$

By taking the real part with respect to i of the above expression and using the inversion formula for Fourier transforms, we get

$$\phi(x;y,z) = -2\kappa \int_{-\infty}^{x} d\xi \; Q(\xi) \int_{0}^{\infty} \frac{dk}{\sqrt{\kappa k}} e^{kz} \cos ky \; \sin\sqrt{\kappa k}(x-\xi)$$

$$+ \int_{-\infty}^{\infty} d\lambda \; e^{-i\lambda x} \; P^*(\lambda) \; e^{\lambda^2 z/\kappa} \cos(\lambda^2 y/\kappa)$$

$$+ \sum_{m=1}^{\infty} \left[q_{2m}(x) \frac{\cos 2m\theta}{r^{2m}} - \frac{q''_{2m}(x)}{\kappa(2m-1)} \frac{\cos(2m-1)\theta}{r^{2m-1}} \right] . \qquad (6.16)$$

The first term in the above expression represents a line of pseudo-impulsive sources placed on the free surface. The limit of this term as y and z tend to zero is singular (see the discussion following equation (4.8)), but since the potential must be analytic everywhere ahead of the bow, therefore $Q(x) \equiv 0$ when $x<0$.

For the purpose of understanding the second term in (6.16) conceptually, it is convenient to make a substitution $t=x/U$, with t interpreted as a time variable. Then the integrand represent a standing wave potential with amplitude proportional to $P^*(\lambda)$ and wave length $2\pi\kappa/\lambda^2$. However, the integral represents the potential due to propagation of an initial disturbance $P(t)$ as discussed by Wehausen and Laitone (1960, p 507).

The sum term in (6.16) represents the so-called wave-free potentials. These functions are singular when $r=0$, therefore we shall require that $q_{2m}(x)=q''_{2m}(x)\equiv 0$, for $x<0$.

It remains now to examine (6.16) more closely and decide what are the unknown quantities that can be obtained from the near-field solution. First of all, we note that there is no physical problem to solve when $x<0$, because the body ceases to exist in this case. From the above discussions the solution ahead of the bow is determined by the second term in (6.16), which include the unknown function $P(x)$. Also, it can be shown that

$$\phi(x;0,0) = P(x) \quad , \quad \text{for} \quad x<0 \; .$$

This means that the function $P(x)$ is definitely related to the disturbance caused ahead of the bow and it can only be determined from matching to the far-field problem. Therefore, the near-field problem can be solved completely for $x>0$ if we know $P(x)$ beforehand. Consequently, the unknown quantities $Q(x)$ and $q_{2m}(x)$ in (6.16) are entirely determined from the near-field solution.

The asymptotic approximation of the near-field problem as $r\to\infty$ while keeping $x/\sqrt{r}$ fixed can easily be obtained from (6.16). Substituting $k=\kappa u^2/r$ in the inner integral of the first term, and $\lambda = \kappa u/\sqrt{r}$ in the second term, then as $r\to\infty$ we get

$$\phi(x;y,z) \sim -\frac{4\kappa}{\sqrt{r}}\int_0^x d\xi\, Q(\xi)\int_0^\infty du\, e^{-\kappa u^2\cos\theta}\cos(\kappa u^2\sin\theta)\,\sin(\kappa u(x-\xi)/\sqrt{r})$$

$$+\frac{\kappa}{\sqrt{r}}\int_{-\infty}^{\infty} du\, e^{-i\kappa ux/\sqrt{r}}\,P^*(\kappa u/\sqrt{r})\,e^{-\kappa u^2\cos\theta}\cos(\kappa u^2\sin\theta)\ . \tag{6.17}$$

7. MATCHING THE BOW FLOW

The inner and outer solutions are matched in a suitable overlap domain in which $x/\sqrt{r} = O(1)$ and $\varepsilon \ll r/L \ll 1$. The limit of the outer solution in the overlap domain is given by equation (4.20), and the limit of the inner solution in the same domain is given by equation (6.17). Therefore, the matching can be accomplished by equating the two expressions.

Consider the first term in both equations. Since the ξ-integration has a variable upper limit x, therefore the first term in (4.20) is equal to the first term in (6.17) if we set

$$m_S(x) = \frac{1}{2} Q(x) \quad . \tag{7.1}$$

It is convenient to match the second term in both (4.20) and (6.17) in the Fourier domain. Thus we obtain

$$P^*(k) = -\,4\pi i \,\mathrm{sgn}(k)\, m_R^*(k) \quad . \tag{7.2}$$

The above relation between P^* and m_R^* is equivalent to a Hilbert transform relationship between P and m_R in the x-domain (see Titchmarch 1948, p. 120). Hence

$$P(x) = 4 \int_{-\infty}^{\infty} \frac{m_R(\xi)}{\xi - x}\, d\xi \quad , \tag{7.3}$$

where the integral must be interpreted as a principal value integral.

The matching of the bow flow is now completed. However, we still need to find the "rapidly" varying source function $m_R(x)$ in order to determine the function $P(x)$ from (7.3) and solve the bow-near-field problem completely. This will be achieved when we consider the next step in the analysis and match the bow flow to the middle-body flow.

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APPENDIX I. ON LINE DISTRIBUTION OF KELVIN SOURCES

Suppose the line of sources lies on the mean free surface $z=0$, along the x-axis, and that z decreases with depth. The y-axis is horizontally at right angles to the x-axis. The co-ordinate system moves with the sources at a constant speed U in the negative x-direction. The source distribution is assumed to extend from $x=-\infty$ to $x=\infty$, and it either vanishes identically outside a finite range or decay rapidly as $|x|\to\infty$. We will require the source density function $m(x)$ to be continuous, but allow its first derivative to be discontinuous at some points.

Determination of the velocity potential.

The velocity potential for this problem was obtained by Tuck (1963), but his results are not in a form suitable for our purpose. An alternative derivation is briefly outlined here, which reduces the results to a form similar to that of Peters (1947).

According to Tuck (1963), the solution of the problem can be expressed in the form

$$\phi(x,y,z) = \phi_1 - 2\int_{-\infty}^{\infty} dk\, e^{-ikx}\, m^*(k)\, k^2 \int_{-\infty}^{\infty} \frac{d\lambda}{\sqrt{k^2+\lambda^2}}\, \frac{\exp(-i\lambda y+z\sqrt{k^2+\lambda^2})}{\kappa\sqrt{k^2+\lambda^2} - k^2} ,$$

where

$$m^*(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dx\, e^{ikx}\, m(x)$$

is the Fourier transform of $m(x)$, and $\kappa=U^2/g$ is the wave number of a plane wave of phase velocity U. The integrals in the above expression must be interpreted in such a way that satisfies the radiation condition as will be seen later. The term ϕ_1 represents the potential due to a line of sources which satisfy the familiar rigid wall free-surface condition, and is given by

$$\phi_1 = -2\int_{-\infty}^{\infty} d\xi\, \frac{m(\xi)}{[(x-\xi)^2+r^2]^{1/2}} , \tag{A1.1}$$

where

$$r = (y^2+z^2)^{1/2}.$$

By use of the Fourier transform convolution theorem, we can write ϕ as

$$\phi = \phi_1 - \frac{1}{\pi} \int_{-\infty}^{\infty} d\xi\, m'(\xi)\, G(x-\xi, y, z) \quad , \tag{A1.2}$$

where $m'(\xi)$ denotes the first derivative of the source function, and G is given by

$$G(x,y,z) = -4 \text{ Im} \int_0^{\infty} d\lambda \cos(\lambda y) \int_0^{\infty} \frac{kdk}{\sqrt{k^2+\lambda^2}} \cdot \frac{\exp(-ikx+z\sqrt{k^2+\lambda^2})}{\kappa\sqrt{k^2+\lambda^2} - k^2} \quad .$$

When the substitution $t=\sqrt{k^2+\lambda^2}$ is made in the integral with respect to k, it becomes

$$G(x,y,z) = 4 \text{ Im} \int_0^{\infty} d\lambda \cos(\lambda y) \int_C dt \frac{\exp(-ix\sqrt{t^2-\lambda^2}+zt)}{t^2 - \kappa t - \lambda^2} \quad . \tag{A1.3}$$

Here the path C is chosen as shown in Figure A-1 so as to avoid the zeros of $t^2-\kappa t-\lambda^2$, namely

$$t_1 = \frac{1}{2} (\kappa + \sqrt{\kappa^2+4\lambda^2}) > \lambda > 0$$

$$t_2 = \frac{1}{2} (\kappa - \sqrt{\kappa^2+4\lambda^2}) < 0 \quad ,$$

and so as to make the integral over t vanishes as $x \to -\infty$.

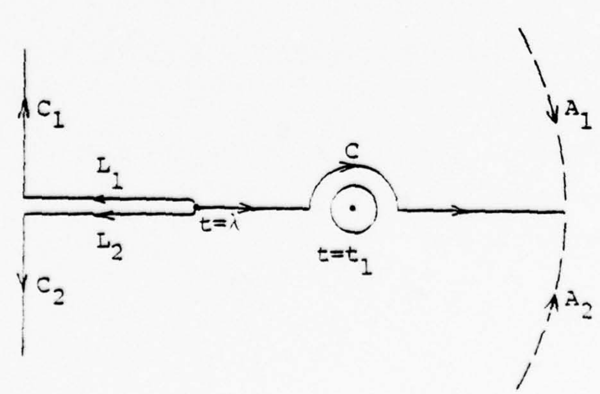

Figure A-1. Indented path of integration C in the cut t-plane

The contour of integration C in (A1.3) may be completed as indicated in the Figure and the final expression for G can easily be obtained by following the work of Peters (1947). Thus without more ado, the potential function ϕ may be expressed as the sum of three parts, i.e.,

$$\phi = \phi_1 + \phi_2 + \phi_3 \quad ,$$

where

$$\phi_2 = -\frac{2}{\pi}\int_{-\infty}^{\infty} d\xi \; m'(\xi) \; \mathrm{sgn}(x-\xi)\int_0^{\infty} du \; e^{-u|x-\xi|} \cdot \mathrm{Re}\int_0^{\pi} d\gamma \; \frac{\exp[iu(y\cos\gamma+z\sin\gamma)]}{u + i\,\kappa\,\sin\gamma} \quad , \tag{A1.4}$$

and

$$\phi_3 = 4\int_{-\infty}^{x} d\xi \; m'(\xi) \; \mathrm{Re}\int_{-\infty}^{\infty} \frac{du}{\sqrt{u^2+1}} \exp\{\kappa z(u^2+1)+i\kappa[(x-\xi)-uy]\sqrt{u^2+1}\} \tag{A1.5}$$

We note that the potentials ϕ_1 and ϕ_2 represent some kind of a local disturbance behavior that decays rapidly at large distances from the sources, while ϕ_3 includes all the wave effects.

Approximation of the potential near to the singular line

We shall evaluate now the asymptotic behavior of each part of the potential ϕ when both y and z become small. The behavior of the term ϕ_1 as $r \to 0$ is well known from similar work on slender-body theory in an unbounded fluid, thus

$$\phi_1 \sim 4\,m(x)\,\log r - 2\int_{-\infty}^{\infty} d\xi \; m'(\xi) \; \mathrm{sgn}(x-\xi) \; \log 2|x-\xi| \quad . \tag{A1.6}$$

The integrals involved in the expression for the term ϕ_2 are bounded and its first approximation near to the singular line can be obtained simply by setting both y and z identically zero in (A1.4), hence

$$\phi_2 \sim -2 \int_{-\infty}^{\infty} d\xi \; m'(\xi) \; \mathrm{sgn}(x-\xi) \int_0^{\infty} du \; \frac{e^{-u|x-\xi|}}{\sqrt{u^2 + \kappa^2}}$$

$$= -\pi \int_{-\infty}^{\infty} d\xi \; m'(\xi) \; \mathrm{sgn}(x-\xi) \; [H_0(\kappa|x-\xi|) - Y_0(\kappa|x-\xi|)] \quad , \qquad \text{(A1.7)}$$

where $H_0(x)$ is Struve function and $Y_0(x)$ is Bessel function of the second kind.

The asymptotic behavior of the terms ϕ_1 and ϕ_2 near the singular line can now be obtained by adding (A1.6) and (A1.7), therefore

$$\phi_1 + \phi_2 \sim 4 \; m(x) \log r - 2 \int_{-\infty}^{\infty} d\xi \; m'(\xi) \; \mathrm{sgn}(x-\xi)$$

$$\left[\log 2|x-\xi| + \frac{\pi}{2} \; H_0(\kappa|x-\xi|) - \frac{\pi}{2} Y_0(\kappa|x-\xi|)\right] \quad . \qquad \text{(A1.8)}$$

The error involved in the approximation of the above expression is bounded uniformly by a function of r that tends to zero as $r \to 0$. The combination "H_0-Y_0" occurring in (A1.8) is well-known to be a monotone decreasing function of its (positive) argument. Furthermore, the kernel of the integral is bounded, since the "log" term precisely cancels the logarithmic behavior of the Y_0 function as $\xi \to x$. Hence the expression (A1.8) is a valid approximation for the complete potentials $\phi_1 + \phi_2$ if the source distribution is continuous and has a piecewise continuous first derivative.

The approximation of the potential ϕ_3 near to the singular line depends generally on how bounded the integral with respect to u in (A1.5) becomes as y and z tend to zero. Ursell (1960) investigated the behavior of such integrals in his study of the wave system generated by a travelling pressure point in the vicinity of its track. He showed that the limits of the integral are non-uniform; in particular, the wave amplitude tends to infinity as we approach the track on the free surface (as $y \to 0$ on $z=0$), but it is finite if we approach the track from below the free surface (if $z \to 0$ on $y=0$). However, Ursell's results cannot be applied directly to our problem because we are dealing with a distribution of singularities and the results will depend on the smoothness of their distribution.

Let us consider the integral with respect to u in the expression (A1.5). According to Ursell's analysis, the asymptotic behavior of this integral is determined by the form of the oscillating term in its integrand. The exact form of this term in our problem depends on the outcome of the ξ-integration which involves the specific details of the source function. However, it is possible to evaluate the significant form of the oscillating term in the integral over u by considering first the asymptotic behavior of the integral with respect to ξ for large values of $|u|$.

Let us consider first the case when $m'(\xi)$ is continuous over the entire range of the ξ-integration, it can be shown that

$$\int_{-\infty}^{x} d\xi \; m'(\xi) \; e^{i\kappa(x-\xi)\sqrt{u^2+1}} = \frac{i\; m'(x)}{\kappa\sqrt{u^2+1}} + O\left(\frac{1}{u^2}\right) ,$$

as $|u|\to\infty$. In this case the integrand reduces to

$$\frac{1}{u^2+1} \; \exp\{\kappa z(u^2+1) - i\kappa y \; u\sqrt{u^2+1}\} \qquad \text{as} \quad |u|\to\infty ,$$

and the integral over u is uniformly convergent as y and z tend to zero. Therefore, the first approximation of ϕ_3 becomes

$$\phi_3 \sim 4 \int_{-\infty}^{x} d\xi \; m'(\xi) \int_{-\infty}^{\infty} \frac{du}{\sqrt{u^2+1}} \cos(\kappa(x-\xi)\sqrt{u^2+1})$$

$$= - 4\pi \int_{-\infty}^{x} d\xi \; m'(\xi) \; Y_0(\kappa(x-\xi)) \quad . \qquad \text{(A1.9)}$$

If this result is added to equation (A1.8) we obtain an expression for the asymptotic behavior of the total potential ϕ near to the singular line which is identical to the usual results of ordinary slender-ship theory, as expected in this case.

On the other hand, if the derivative of the source function is discontinuous at some point x_0 , then

$$\int_{-\infty}^{x} d\xi \; m'(\xi) \; e^{i\kappa(x-\xi)\sqrt{u^2+1}} = \frac{i}{\kappa\sqrt{u^2+1}} \left[m'(x) - \Delta m'(x_0) \; e^{i\kappa(x-x_0)\sqrt{u^2+1}} \right] + O\left(\frac{1}{u^2}\right)$$

as $|u|\to\infty$; where $\Delta m'(x_0)$ denotes the jump in the derivative at the point x_0 . In this case the oscillating term which determines the asymptotic behavior of the integral with respect to u for small values of y and z must include the exponential term of the above expression. Therefore, we can write the integral over u in (A1.5) as follows:

$$I \equiv \mathrm{Re} \int_{-\infty}^{\infty} \frac{du}{\sqrt{u^2+1}} \exp[\kappa z(u^2+1) + i\kappa(x_0-\xi)\sqrt{u^2+1}]$$

$$\exp\{i\kappa[(x-x_0)-u|y|]\sqrt{u^2+1}\} . \qquad \text{(A1.10)}$$

This integral is similar to the one considered by Ursell (1960) and the subsequent analysis will follow his procedure.

Let us apply the steepest descent method to the integral (A1.10) using the oscillating term which becomes dominant as $|u|\to\infty$. The path of integration can be deformed into two paths of steepest descent through the two saddle points, which are the roots of the equation

$$\frac{d}{du}\{[(x-x_0) - u|y|]\sqrt{1+u^2}\} = 0$$

i.e. the points

$$u_{\pm} = \frac{1}{4} \quad \frac{(x-x_0)}{|y|} \pm \left[\frac{(x-x_0)^2}{y^2} - 8\right]^{1/2} ,$$

which are real if $(x-x_0)^2>8y^2$. The paths of steepest descent C_- and C_+ in the cut u-plane are given by Ursell and shown schematically in Figure A-2.

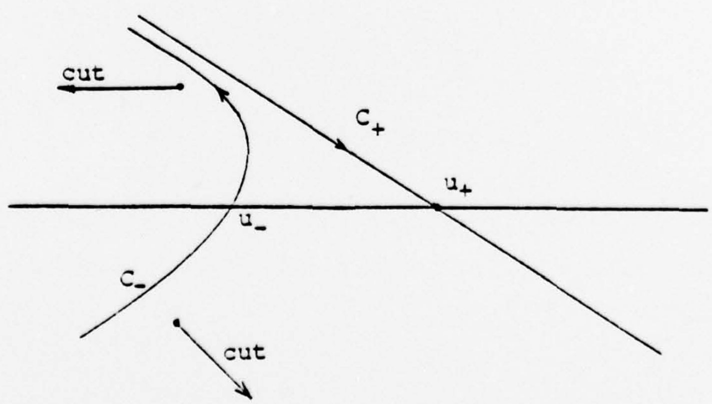

Figure A-2. Paths cut of steepest descent in the cut u-plane

Let us consider the behavior of the integrals along C_- and C_+ as y and z tend to zero, provided that $y/(x-x_0) \ll 1$. It is easy to see that the integrand tends to zero uniformly as $|u| \to \infty$ along C_-, thus

$$\int_{C_-} \sim \int_{-\infty e^{i\pi/4}}^{\infty e^{i3\pi/4}} \frac{du}{\sqrt{u^2+1}} e^{i\kappa(x-\xi)\sqrt{u^2+1}}$$

This integral represents contributions from the transverse wave system and it can be expressed in terms of Y_0-Bessel function, hence

$$\mathrm{Re} \int_{C_-} \sim -\pi\, Y_0(\kappa(x-\xi)) \quad . \tag{A1.11}$$

We note that the contribution of the integral along C_- to the total potential ϕ is equal to the contribution from the part ϕ_3 in the case of a source distribution with continuous first derivative. But here we still have to evaluate the behavior of the integral along C_+ near to the singular line which represent contributions of the divergent wave system. It is obvious that the saddle point at u_+ tends to infinity as $|y| \to 0$; thus the contour C_+ moves off to infinity. To avoid this difficulty, we introduce the new variable $v = u|y|/(x-x_0)$, in terms of which

$$\int_{C_+} = \int_{\infty e^{i3\pi/4}}^{\infty e^{-i\pi/4}} \frac{dv}{\sqrt{v^2+\delta^2}} \exp[\kappa z(v^2+\delta^2)/\delta^2 + i\kappa(x_0-\xi)\sqrt{v^2+\delta^2}/\delta]$$

$$\exp\left[i\kappa \frac{(x-x_0)^2}{|y|} (1-v)\sqrt{v^2+\delta^2}\right] \quad , \tag{A1.12}$$

where $\delta = |y|/(x-x_0) \ll 1$. The saddle point is now at $v_+ = \frac{1}{4}(1+\sqrt{1-8\delta^2})$, which remains finite as $|y| \to 0$.

It is evident that the asymptotic behavior of the above integral depends on the parameter $\kappa(x-x_0)^2/|y|$. If the value of this parameter is large which corresponds to $\kappa(x-x_0) = O(1)$, then we can apply the ordinary method of steepest descent (see, for example, Jeffreys & Jeffreys 1946, p 503) to get

$$\int_{C_+} \sim 2\left(\frac{2\pi|y|}{\kappa(x-x_0)^2}\right)^{1/2} \exp\left[\kappa \frac{(x-x_0)^2}{4|y|} - \frac{\pi}{4}\right] \exp[\kappa z/4\delta^2 + i\kappa(x_0-\xi)/2\delta] \quad .$$

Therefore, when $\kappa(x-x_0)=O(1)$, the first approximation of the potential ϕ_3 near to the singular line becomes

$$\phi_3 \sim -4\pi \int_{-\infty}^{x} d\xi\, m'(\xi)\, Y_0(\kappa(x-\xi)) + O(|y|^{3/2}) \quad . \tag{A1.13}$$

This result is identical to the one obtained in the case of a source distribution with continuous first derivative. Therefore, ordinary slender-ship theory results are still valid in the case of a source distribution with discontinuous first derivative at large distances from the point of discontinuity. However, if the parameter $\kappa(x-x_0)^2/|y|=O(1)$ the integral (A1.12) can be evaluated when $\delta\to 0$ (the details are omitted), and the potential ϕ_3 becomes

$$\phi_3 = -4\pi \int_{-\infty}^{x} d\xi\, m'(\xi)\, Y_0(\kappa(x-\xi)) + 4\sqrt{\pi}\, \mathrm{Im}\left[\Omega \int_{-\infty}^{x} d\xi\, m(\xi)\, e^{-\Omega^2(x-\xi)^2} \mathrm{erf}[-i\Omega(x-\xi)]\right] \tag{A1.14}$$

where erf denotes the error function as defined by Abramowitz and Stegun (1965), and

$$\Omega = [\kappa/(-z+i|y|)]^{1/2} \quad .$$

We note that the first term in (A1.14) is now small compared to the second term, because $|\Omega|=O(r^{-1/2})$. Thus the contribution of the divergent wave system dominates the flow field near the singular line in this case.

APPENDIX II. ERROR ESTIMATION

It will be shown here that under certain sufficient conditions the approximations given in section 4 differ from the complete potentials by an error term which is $O(\varepsilon^{1/2})$. The regularity conditions required in the analysis are:

(i) The source function $m(x)$ is continuous and has a piecewise continuous first derivative everywhere.

(ii) The function $m(x)$ is absolutely integrable over the whole x-axis.

A. Estimation of the error E_1

The error term E_1 represents the difference between the complete potential given by (4.1) and its approximation (4.2). Thus we get

$$E_1(x;r,\theta) = \frac{4\kappa}{\sqrt{r}} \int_{-\infty}^{\infty} d\ell\, e^{-i\ell x/\sqrt{r}}\, m^*(\ell/\sqrt{r})\ \mathrm{Re} \int_0^{\infty} d\lambda\, \frac{\exp(-i|\ell|\lambda\cos\theta)}{\kappa\lambda + i|\ell|}$$
$$\left\{\exp(-|\ell|\lambda\sin|\theta|) - \frac{\lambda}{\sqrt{\lambda^2+r}} \exp(-|\ell|\sqrt{\lambda^2+r}\,\sin|\theta|)\right\} . \qquad \text{(A2.1)}$$

First, let us investigate the x-derivative of the function E_1. From (A2.1), the Fourier transform of the function $D_1 \equiv \partial E_1/\partial x$ in terms of the variable $\ell/\sqrt{r}$ can be written in the form

$$D_1^*(\ell/\sqrt{r};r,\theta) = -i\,\frac{4\kappa}{r}\,\ell\, m^*(\ell/\sqrt{r}) \int_0^{\infty} d\lambda\ \mathrm{Re}\,\frac{\exp(-i|\ell|\lambda\cos\theta)}{\kappa\lambda + i|\ell|}$$
$$\left[\exp(-|\ell|\lambda\sin|\theta|) - \frac{\lambda}{\sqrt{\lambda^2+r}} \exp(-|\ell|\sqrt{\lambda^2+r}\,\sin|\theta|)\right] .$$

By use of the first mean-value theorem (see, for example, Jeffreys & Jeffreys, 1946, pp 50), we get

$$D_1^* \leq -i\,\frac{4\kappa}{r}\,\ell\, m^*(\ell/\sqrt{r})\ \mathrm{Re}\left\{\frac{\exp(-i|\ell|\lambda_0\cos\theta)}{\kappa\lambda_0 + i|\ell|}\right\}$$
$$\int_0^{\infty} d\lambda \left[\exp(-|\ell|\lambda\sin|\theta|) - \frac{\lambda}{\sqrt{\lambda^2+r}} \exp(-|\ell|\sqrt{\lambda^2+r}\,\sin|\theta|)\right] ,$$

where λ_0 denotes the value of λ at which the real part of the function attains its maximum value. The λ-integration can be obtained analytically, thus

$$|D_1^*| \leq \frac{4\kappa}{r} |m^*(\ell/\sqrt{r})| \left|\frac{\exp(-i|\ell|\lambda_0 \cos\theta)}{\kappa\lambda_0 + i|\ell|}\right| \left|1 - e^{-|\ell|\sqrt{r}\sin|\theta|}\right| \Big/ \sin|\theta|$$

$$\leq \frac{4\kappa}{\sqrt{r}} \left|m^*(\ell/\sqrt{r})\right| \quad .$$

We can write

$$|D_1| \leq 4\kappa \int_{-\infty}^{\infty} dk \ |m^*(k)|$$

Therefore, it follows that

$$\frac{\partial E_1}{\partial x} = O(1) \quad , \tag{A2.2}$$

provided that $m^*(k)$ is absolutely integrable, which is a trivial consequence of the stated regularity conditions on $m(x)$. However, in the bow region $x=O(\varepsilon^{1/2})$, consequently

$$\frac{\partial E_1}{\partial x} = O(E_1 \ \varepsilon^{-1/2}) \ . \tag{A2.3}$$

From (A2.2) and (A2.3) we finally obtain

$$E_1 = O(\varepsilon^{1/2}) \ . \tag{A2.4}$$

B. <u>Estimation of the error E_2</u>

The expression for E_2 represents the difference between the complete potential in (4.5) and its approximation in (4.6). Thus

$$E_2 = 4\pi i \frac{\kappa}{\sqrt{r}} \int_{-\infty}^{\infty} d\ell \ e^{-i\kappa x/\sqrt{r}} \ m^*(\kappa\ell/\sqrt{r}) \ e^{-\kappa\ell^2\cos\theta}$$

$$\left\{\mathrm{sgn}(\ell) \ e^{i\kappa\ell|\ell|\sin|\theta|} - \frac{\ell}{\sqrt{\ell^2-r}} \ e^{i\kappa\ell\sqrt{\ell^2-r}\ \sin|\theta|}\right\} \tag{A2.5}$$

where the integral is taken along the upper side of the branch cut between $\ell=\pm\sqrt{r}$. The error term E_2 can be expressed as the sum of two parts, namely

$$E_2 = E_3 + E_4$$

where the functions E_3 and E_4 may be reduced to the following forms:

$$\begin{aligned} E_3 = {} & 8\pi\kappa \,\mathrm{Im} \int_0^\infty d\gamma\, e^{-i\kappa x \mathrm{ch}\gamma}\, \mathrm{ch}\gamma\, m^*(\kappa \mathrm{ch}\gamma) \\ & \exp(-\kappa r \mathrm{ch}^2\gamma e^{-i|\theta|}) \{\exp(-i\kappa|y|\mathrm{ch}\gamma e^{-\gamma})-1\} \\ & -8\pi\kappa \,\mathrm{Re} \int_0^{\pi/2} d\gamma\, e^{-i\kappa x \cos\gamma} \cos\gamma \;\; m^*(\kappa\cos\gamma) \\ & \exp(-\kappa r\cos^2\gamma e^{-i|\theta|})\{\exp(-i\kappa|y|\cos\gamma\; e^{-i\gamma})-1\} \; ; \end{aligned} \tag{A2.6}$$

and

$$\begin{aligned} E_4 = {} & 8\pi\kappa \,\mathrm{Im} \int_0^\infty d\gamma\, e^{-i\kappa x \mathrm{ch}\gamma}\, m^*(\kappa \mathrm{ch}\gamma)\; e^{-\gamma} \;\; \exp(-\kappa r \mathrm{ch}^2\gamma e^{-i|\theta|}) \\ & -8\pi\kappa \,\mathrm{Re} \int_0^{\pi/2} d\gamma\, e^{-i\kappa x \cos\gamma}\, m^*(\kappa\cos\gamma)\; e^{-i\gamma} \;\exp(-\kappa r\cos^2\gamma e^{-i|\theta|}) \; . \end{aligned} \tag{A2.7}$$

In deriving the above expressions, we have made use of the fact that $m^*(-k)=\overline{m^*(k)}$ since $m(x)$ is real.

Let us consider first the expression for E_3. By use of the first mean-value theorem, we can show that the absolute value of the functions in the { } brackets are $\leq\kappa|y|$. Thus

$$|E_3| \leq 8\pi\,\kappa^2\,|y| \left\{ \int_0^\infty d\gamma\; \mathrm{ch}\gamma\; |m^*(\kappa \mathrm{ch}\gamma)| + \int_0^{\pi/2} d\gamma\; \cos\gamma\; |m^*(\kappa\cos\gamma)| \right\} .$$

Since $m(x)$ is assumed to be continuous with piecewise continuous first derivative, hence $|k\; m^*(k)|$ is certainly bounded for all k by the Riemann-Lebesgue lemma, and

$$|m^*(k)| \leq \frac{M}{|k|} \qquad \text{(A2.8)}$$

for some finite positive number M. Putting $\text{ch}\gamma = \text{sh}\gamma + e^{-\gamma}$ and use (A2.8) in the expression for $|E_3|$, we get

$$|E_3| \leq 8\pi\kappa\, |y| \left\{ \int_{-\infty}^{\infty} dk \;\; m^*(k) \;\; + \left(1 + \frac{\pi}{2}\right) M \right\}$$

Therefore, we may conclude that

$$E_3 = O(|y|) \quad . \qquad \text{(A2.9)}$$

In order to estimate the error term E_4, we shall consider first its derivative with respect to x. From (A2.9), we obtain

$$\left|\frac{\partial E_4}{\partial x}\right| \leq 8\pi\kappa \left\{ \int_0^{\infty} d\gamma \; |\kappa\text{ch}\gamma| \; |m^*(\kappa\text{ch}\gamma)| \;\; e^{-\gamma} \right.$$

$$\left. + \int_0^{\pi/2} d\gamma \; |\kappa\cos\gamma| \;\; |m^*(\kappa\text{ch}\gamma)| \right\} \quad .$$

Using (A2.8), hence

$$\left|\frac{\partial E_4}{\partial x}\right| \leq 8\pi\kappa\, M \left(1 + \frac{\pi}{2}\right) \quad .$$

It follows that

$$\frac{\partial E_4}{\partial x} = O(1) \quad ,$$

for the same reasons that led to (A2.3), thus

$$E_4 = O(\varepsilon^{1/2}) \quad . \qquad \text{(A2.10)}$$

Finally, the estimate of the error term E_2 can be obtained from (A2.9) and (A2.10), hence

$$E_2 = O(\varepsilon^{1/2}) \quad . \qquad \text{(A2.11)}$$

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3 Commander
Naval Ocean Systems Center
San Diego, CA 92152
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1 Library
Naval Underwater Systems Center
Newport, RI 02840

1 Dept. of Transportation
Library TAD-491.1
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Washington, DC 20590

1 Charleston Naval Shipyard
Technical Library
Naval Base
Charleston, SC 29408

1 Norfolk Naval Shipyard
Technical Library
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Philadelphia, PA 19112
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1 Portsmouth Naval Shipyard
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1 Puget Sound Naval Shipyard
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1 Mare Island Naval Shipyard
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2 U.S. Naval Academy
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1 Naval Postgraduate School
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1 Study Center
National Maritime Research
Center
U.S. Merchant Marine Academy
Kings Point, L.I., NY 11024

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Kings Point, L.I., NY 11024
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1 Library
The Pennsylvania State University
Applied Research Laboratory
PO Box 30
State College, PA 16801

1 Bolt, Beranek & Newman
50 Moulton Street
Cambridge, MA 02138
Attn: Library

1 Bethlehem Steel Corporation
Center Technical Division
Sparrows Point Yard
Sparrows Point, MD 21219

1 Bethelehem Steel Corporation
25 Broadway
New York, NY 10004
Attn: Library (Shipbuilding)

1 Mr. V. Boatwright, Jr.
R & D Manager
Electric Boat Division
General Dynamics Corporation
Groton, CT 06340

1 Gibbs & Cox, Inc.
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New York, NY 10006
Attn: Technical Info. Control

1 Hydronautics, Inc.
Pindell School Road
Howard County
Laurel, MD 20810
Attn: Library

2 McDonnell Douglas Aircraft Co.
3855 Lakewood Blvd
Long Beach, CA 90801
Attn: J. Hess
T. Cebeci

1 Lockheed Missiles & Space Co.
PO Box 504
Sunnyvale, CA 94088
Attn: Mr. R.L. Waid, Dept. 57-74
Bldg. 150, Facility 1

1 Newport News Shipbuilding &
Dry Dock Company
4101 Washington Avenue
Newport News VA 23607
Attn: Technical Library Dept.

1 Nielsen Engineering & Research Inc.
510 Clude Avenue
Mountain View, CA 94043
Attn: Mr. S. Spangler

1 Oceanics, Inc.
Technical Industrial Park
Plainview, L.I., NY 11803

1 Society of Naval Architects
and Marine Engineers
One World Trade Center, Suite 1369
New York, NY 10048
Attn: Technical Library

1 Sun Shipbuilding & Dry Dock Co.
Chester, PA 19000
Attn: Chief Naval Architect

1 Sperry Systems Management Division
Sperry Rand Corporation
Great Neck, NY 11020
Attn: Technical Library

1 Stanford Research Institute
Menlo Park, CA 94025
Attn: Library G-021

2 Southwest Research Institute
PO Drawer 28510
San Antonio, TX 78284
Attn: Applied Mechanics Review
Dr. H. Abramson

1 Tracor, Inc.
6500 Tracor Lane
Austin, TX 78721

1 Mr. Robert Taggart
9411 Lee Hgwy, Suite P
Fairfax, VA 22031

1 Ocean Engr Department
Woods Hold Oceanographic Inst.
Woods Hole, MA 02543

1 Worcester Polytechnic Inst.
Alden Research Laboratories
Worcester, MA 01609
Attn: Technical Library

1 Applied Physics Laboratory
University of Washington
1013 N.E. 40th Street
Seattle, WA 98105
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1 University of Bridgeport
Bridgeport, CT 06602
Attn: Dr. E. Uram

4 University of California
Naval Architecture Department
College of Engineering
Berkeley, CA 94720
Attn: Library
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Prof. J. Paulling
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3 California Institute of Technology
Pasadena, CA 91109
Attn: Aeronautics Library
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Dr. A.J. Acosta

2 Florida Atlantic University
Ocean Engineering Department
Boca Raton, FL 33432
Attn: Technical Library
Dr. S. Dunne

1 University of Hawaii
Department of Ocean Engineering
2565 The Mall
Honolulu, HI 96822
Attn: Library

1 Institute of Hydraulic Research
The University of Iowa
Iowa City, IA 52240
Attn: Library

3 Department of Ocean Engineering
Massachusetts Institute of Technology
Cambridge, MA 02139
Attn: Prof. P. Mandel
Prof. M. Abkowitz
Dr. J. Newman

1 Engineering Technical Reports
Room 10-500
Massachusetts Institute of Technology
Cambridge, MA 02139

1 St. Anthony Falls Hydraulic Laboratory
University of Minnesota
Mississippi River at 3rd Avenue SE
Minneapolis, MN 55414
Attn: Dr. Roger Arndt

1 College of Engineering
University of Notre Dame
Notre Dame, IN 46556
Attn: Engineering Library

3 Davidson Laboratory
Stevens Institute of Technology
711 Hudson Street
Hoboken, NJ 07030
Attn: Library
Mr. J. Breslin
Mr. S. Tsakonas

2 Stanford University
Stanford, CA 94305
Attn: Engineering Library
Dr. R. Street

3 Webb Institute of Naval Architecture
Crescent Beach Road
Glen Cove, LI, NY 11542
Attn: Library
Prof. E.V. Lewis
Prof. L.W. Ward

1 Applied Research Laboratory
PO Box 30
State College PA 16801
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Garfield Thomas Water Tunnel

1 Dr. Michael E. McCormick
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1 Dr. Douglas E. Humphreys (Code 794)
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